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LETTER TO THE EDITOR

The Hall factor of hot-electron transport in non-parabolic Kane bands

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Abstract. Balance equations of hot-carrier transport have recently been developed for electrons in an arbitrary energy band in crossed electric and magnetic fields. It is shown that these equations provide a very convenient tool to deal with semiclassical magneto-hot-electron transport in non-parabolic systems. As an example, Hall factors of narrow-gap semiconductors with Kane band structures are obtained by directly calculating the γ-coefficients in the equations. The predictions, which are sensitive to band non-parabolicity, are compared with an earlier experiment and Monte Carlo analysis.

The Lei–Ting balance equation approach [1] has been shown to be a useful tool for studying electron transport in three- and quasi-two-dimensional semiconductor systems [2–11]. It provides a much more tractable method to analyse carrier conduction under the influence of crossed magnetic and electric fields than direct solution of the Boltzmann equation or use of the Kubo formula [2, 10]. Although the original balance equations were developed for electrons moving in a parabolic band, they have recently been extended to systems with a general energy dispersion in an electric field [12] and in crossed magnetic and electric fields [13]. The purpose of this letter is to point out that these newly extended balance equations provide a very convenient tool to deal with semiclassical magneto-hot-electron transport in a non-parabolic system. As an example, we apply them to calculate the Hall factor of narrow-gap semiconductors with Kane band structure.

Consider N interacting electrons moving within an isotropic, non-parabolic Kane band [15]:

$$\varepsilon(k) = \frac{1}{2\alpha}[(1 + 4\alpha k^2/2m)^{1/2} - 1] \tag{1}$$

where k is the wavevector, m is the electron effective mass at the conduction band bottom of the narrow-gap semiconductor, and

$$\alpha = \frac{1}{\varepsilon_g}(1 - m/m_e)^2 \tag{2}$$

is the non-parabolicity coefficient; ε_g being the energy gap between the conduction and valence bands.

In the balance equation method the transport state of a many-electron system under the influence of an electric field in the x–y plane, $E = (E_x, E_y, 0)$, and a magnetic field in

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the z-direction, $B = (0, 0, B)$, is characterized by a centre-of-mass momentum $P_d \equiv Np_d$, $p_d = (p_x, p_y, 0)$, and an electron temperature T_e. The average velocity of the centre of mass, or the average drift velocity of electrons, $v_d = (v_x, v_y, 0)$, is given by

$$v_d = \frac{2}{N}\sum_k \nabla\varepsilon(k) f(\varepsilon(k - p_d),\ T_e) \tag{3}$$

where $f(\varepsilon,\ T_e) \equiv 1/[\exp(\varepsilon - \mu)/T_e + 1]$ is the Fermi function at temperature T_e with μ being the chemical potential to be determined by the total number of carriers:

$$N = 2\sum_k f(\varepsilon(k),\ T_e). \tag{4}$$

The centre of mass is a single particle, having charge Ne and inverse effective mass tensor K/N, $K_{ij} \equiv 1/m^*_{ij}$ $(i, j = x, y, z)$, given by

$$K = \frac{2}{N}\sum_k \nabla\cdot\nabla\varepsilon(k) f(\varepsilon(k - p_d),\ T_e). \tag{5}$$

In addition, to describe the centre-of-mass motion in this crossed magnetic and electric field configuration, we need four more dimensionless coefficients, $\gamma_{y,xx}$, $\gamma_{x,yy}$, $\gamma_{x,xy}$, $\gamma_{y,xy}$, defined by $(a,\ b,\ c = x,\ y)$

$$\gamma_{a,bc} = \frac{\langle \varepsilon'_a \varepsilon''_{bc}\rangle}{\langle \varepsilon'_a\rangle\langle \varepsilon''_{bc}\rangle}. \tag{6}$$

Here $\varepsilon'_a \equiv \partial\varepsilon(k)/\partial k_a$, $\varepsilon''_{ab} \equiv \partial^2\varepsilon(k)/\partial k_a \partial k_b$ and the bracket $\langle ...\rangle$ stands for the average

$$\langle ...\rangle = \frac{2}{N}\sum_k ... f(\varepsilon(k - p_d),\ T_e). \tag{7}$$

The equations of motion of the centre of mass in crossed magnetic and electric fields take the form [14]

$$\frac{dv_x}{dt} = \frac{eE_x}{m^*_{xx}} + \frac{eE_y}{m^*_{xy}} + \frac{eBv_y}{m^*_{xx}}\gamma_{y,xx} - \frac{eBv_x}{m^*_{xy}}\gamma_{x,xy} + A_x \tag{8}$$

$$\frac{dv_y}{dt} = \frac{eE_x}{m^*_{yx}} + \frac{eE_y}{m^*_{yy}} + \frac{eBv_y}{m^*_{yx}}\gamma_{y,xy} - \frac{eBv_x}{m^*_{yy}}\gamma_{x,yy} + A_y. \tag{9}$$

These two equations, together with the energy balance equation

$$\frac{dh_e}{dt} = eE_x v_x + eE_y v_y - W \tag{10}$$

form a complete set of equations for the determination of parameters p_x, p_y and T_e, and then all the major physical quantities (such as the drift velocities v_x and v_y) of transport in crossed magnetic and electric fields. In the above equations h_e is the average electron energy, $A = (A_x, A_y, 0)$ is the frictional acceleration due to impurity (A_i) and phonon (A_p) scatterings, and W is the energy-loss rate per carrier from the electron system to the phonon system [14].

Note that when both momentum components p_x and p_y (thus both drift velocity components v_x and v_y) are not zero, the non-diagonal elements of the inverse effective mass tensor, $1/m^*_{xy}$ and $1/m^*_{yx}$, are finite. However, choosing the x-axis to be the velocity direction, $v_d = (v_d, 0, 0)$ and $p_d = (p_d, 0, 0)$, we have $1/m^*_{xy} = 1/m^*_{yx} = 0$, and the steady-state force balance equations can be reduced to

$$\frac{eE_x}{m^*_{xx}} + A_x = 0 \tag{11}$$

$$\frac{eE_y}{m^*_{yy}} - \frac{eBv_d}{m^*_{yy}}\gamma_{x,yy} = 0. \tag{12}$$

This set of equations facilitates determination of the steady-state Hall resistivity under hot-electron conduction. Since the current density (in the x-direction) is given by

$$j_x = nev_d \tag{13}$$

where n is the electron number density and e is the electron charge. The (non-linear) longitudinal resistivity, defined as $\rho_{xx} \equiv E_x/j_x$, is obtained from equation (11):

$$\rho_{xx} = -\frac{m^*_{xx}A_x}{ne^2v_d}. \tag{14}$$

On the other hand, the electric field along the y-direction is determined by equation (12):

$$E_y = Bv_d\gamma_{x,yy}. \tag{15}$$

This yields the non-linear transverse (Hall) resistivity

$$\rho_{yx} \equiv \frac{E_y}{j_x} = \frac{\gamma_{x,yy}}{ne}B \tag{16}$$

and the Hall coefficient in this configuration

$$R_H = \frac{\gamma_{x,yy}}{ne} \tag{17}$$

or the Hall factor

$$\gamma = \gamma_{x,yy}. \tag{18}$$

Apparently, it is independent of the direction of the current flow for this isotropic Kane band. However, in addition to the energy dispersion of the system, the Hall factor depends on the centre-of-mass momentum p_d and the electron temperature T_e, and thus depends on the applied electric field, the scatterings in the system, and the lattice temperature, as well as the magnetic field. In a parabolic band, $\nabla \cdot \nabla\varepsilon(k) = \mathcal{I}/m$ (constant), all the γ-coefficients defined by equation (6), and thus the Hall factor γ, are unity under both linear and non-linear transport for an arbitrary electron density at low and high temperatures. Therefore, any deviation of γ from unity must result from non-parabolicity.

To see how the Hall factor changes with temperature, we plot in figures 1 and 2 the weak electric field ($z_d \sim 0$) γ as a function of electron temperature T_e for non-parabolic Kane bands with different non-parabolicity coefficient and band bottom effective mass for varying electron density. Figure 1 ($\alpha = 0.613\,\mathrm{eV}^{-1}$ and $m = 0.067m_e$) corresponds to an n-GaAs Kane band. Figure 2 ($\alpha = 4.32\,\mathrm{eV}^{-1}$ and $m = 0.0138m_e$) corresponds to an n-InSb Kane band. For comparison we also show the corresponding γ for the respective Kane band with $\alpha = 0.01\,\mathrm{eV}^{-1}$ at $n = 2 \times 10^{14}\,\mathrm{cm}^{-3}$ (dashed line in each figure). Such a weak non-parabolicity implies that it represents an almost parabolic system. Indeed, for this case, $\gamma \simeq 1$ except in the highest-temperature region. In strictly parabolic bands, of course, $\gamma = 1$ is independent of temperature. Since the deviation of a Kane band from the corresponding parabolic band is proportional to the square of the wavevector, k, the non-parabolic effect should appear to be stronger when higher wavevector states are occupied, i.e. at higher electron density or at higher electron temperature. For sufficiently low electron density and sufficiently low electron temperature a Kane band should behave parabolically. This is indeed the case, as can be seen in figures 1 and 2, where for the lowest electron density shown in these figures, $n = 2 \times 10^{14}\,\mathrm{cm}^{-3}$, the value of γ always approaches a limit very close to unity.

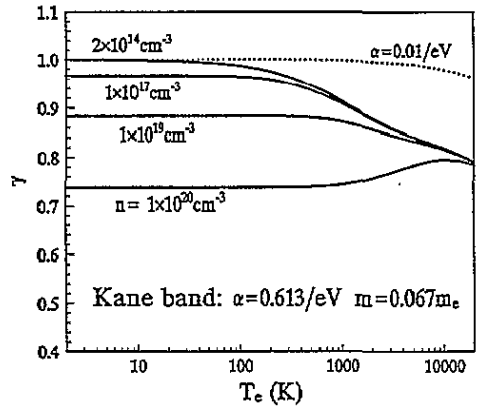

Figure 1. Temperature dependence of the Hall factor γ of an electron system with n-GaAs Kane band ($\alpha = 0.613\,\mathrm{eV}^{-1}$ and $m = 0.067m_e$), having different electron densities n. The dashed curve indicates the corresponding γ for a Kane band with $\alpha = 0.01\,\mathrm{eV}^{-1}$ and $m = 0.067m_e$ at $n = 2 \times 10^{14}\,\mathrm{cm}^{-3}$.

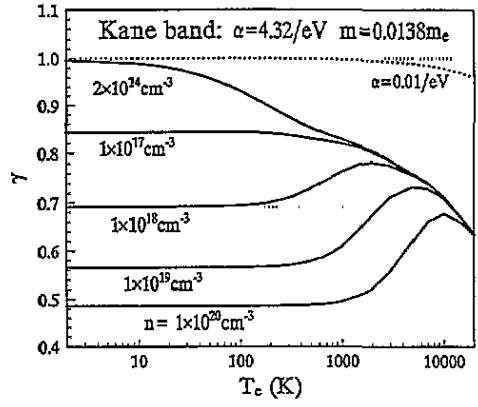

Figure 2. Temperature dependence of the Hall factor γ of an electron system with n-InSb Kane band ($\alpha = 4.32\,\mathrm{eV}^{-1}$ and $m = 0.0138m_e$), having different electron densities n. The dashed curve indicates the corresponding γ for a Kane band with $\alpha = 0.01\,\mathrm{eV}^{-1}$ and $m = 0.0138m_e$ at $n = 2 \times 10^{14}\,\mathrm{cm}^{-3}$.

The Hall factor, γ, introduced here depends on the momentum shift p_d, thus γ changes with changing the strength of the current flow in the system. For convenience the momentum shift will be representated by a dimensionless quantity

$$z_d \equiv \sqrt{\frac{2\alpha}{m}}\,p_d. \tag{19}$$

In figures 3 and 4 we plot the calculated γ as a function of electron temperature T_e for several different values of z_d ranging from 0–5 for an n-InAs Kane band ($\alpha = 2.73\,\mathrm{eV}^{-1}$ and $m = 0.0138m_e$) at electron density $n = 1 \times 10^{19}\,\mathrm{cm}^{-3}$ (figure 3) and $n = 2 \times 10^{14}\,\mathrm{cm}^{-3}$ (figure 4).

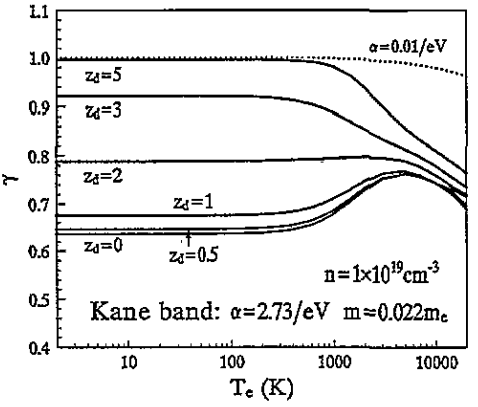

Figure 3. Calculated Hall factor γ as a function of electron temperature T_e at different given momentum shift: $z_d = 0$, 0.5, 1.0, 2.0, 3.0 and 5.0, for an n-InAs Kane band ($\alpha = 2.73\,\mathrm{eV}^{-1}$ and $m = 0.0138m_e$) with electron density $n = 1 \times 10^{19}\,\mathrm{cm}^{-3}$.

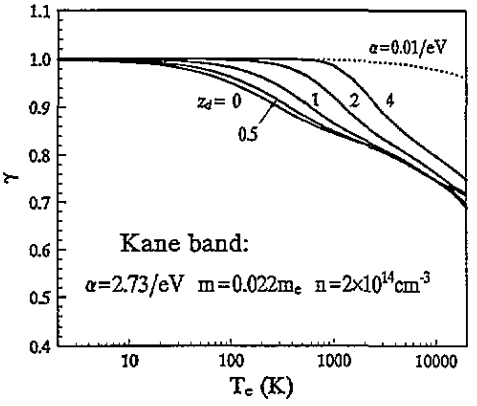

Figure 4. Calculated Hall factor γ as a function of electron temperature T_e at different given momentum shift: $z_d = 0$, 0.5, 1.0, 2.0, 3.0 and 5.0, for an n-InAs Kane band ($\alpha = 2.73\,\mathrm{eV}^{-1}$ and $m = 0.0138m_e$) with electron density $n = 2 \times 10^{14}\,\mathrm{cm}^{-3}$.

There were many experimental and theoretical investigations on hot-electron galvanomagnetic conductions in narrow-gap semiconductors. As examples we would like to mention the experimental measurement of Alberga [15] and the Monte Carlo calculation of Warmenbol *et al* [16] on the Hall effect of n-type InSb. Both the experiment and the theory report a Hall factor decreasing by about 10% with increasing the applied electric field from 10 V cm^{-1} to 400 V cm^{-1}. The temperature dependence of γ exhibited in the $n = 2 \times 10^{14}\,\mathrm{cm}^{-3}$ curve in figure 2 is in agreement with this finding. As a matter of fact, numerical analysis of the steady-state balance equations determines that at an applied field of 400 V cm^{-1} the momentum shift z_d is less than 0.1, for an n-InSb system with polar optic phonon and impurity scatterings, and with electron density n equal to the impurity density $n_i = 1 \times 10^{16}\,\mathrm{cm}^{-3}$. Therefore, up to this electric field strength, the Hall factor γ is essentially the same as that of $z_d = 0$ and the field variation of γ results almost solely

from the electron temperature, which increases from 77 K to about 400 K with an increase in the field strength from 10 to 400 V cm^{-1}. However, the Monte Carlo investigation of [16] yielded a Hall factor, which is larger than 1 at low electric fields and becomes 1 at higher electric fields, and is thus in disagreement with the present result. For a Kane band, our prediction that γ is nearly 1 at low fields and less than 1 at higher fields, is physically more reasonable. Since at such a small electron density only states in a very small region of the k-space around the Kane band bottom are occupied at 77 K, then in and low electric fields, the system behaviour should be close to a parabolic band for which γ is unity.

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